An Algorithm for Generating Log-Aesthetic Curved Surfaces and the Development of a Curved Surfaces Generation System using VR

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Abstract: In the field of industrial design, designers create models with computer-aided design (CAD) and evaluate the models using virtual reality (VR) systems. This evaluation requires precise models in the early phases of modeling. However, it is too difficult for designers to control key-lines as they become aesthetic curved surfaces by CAD. Therefore, modelers must modify key-lines of the clay models that are made from the CAD data. These modifications require a great deal of labor.

This study presents an algorithm for generating “log-aesthetic curved surfaces” that are needed to create precise models using CAD. First, we used our analysis method to clarify characteristics of lines of curvature on several curved surfaces of natural products and crafts. This analysis method extracts the lines of curvature on the curved surfaces and draws a “logarithmic curvature histogram” and a “logarithmic torsion histogram” from the radii of curvature and radii of torsion of the lines of curvature. We showed that the lines of curvature had characteristics of a “log-aesthetic space curve” and classified the curved surfaces into three types on the basis of our results.

Second, we extracted several parameters for controlling characteristics of the log-aesthetic space curves and defined a log-aesthetic curved surface using the log-aesthetic space curves made from these parameters. This log-aesthetic space curve is made from the logarithmic curvature histogram and the logarithmic torsion histogram. The log-aesthetic curved surface is made from one guide line and two base lines. The base lines are the log-aesthetic space curves, and the curvature and torsion change between the two base lines are controlled by a “control line of distribution of curvature and torsion (CLDCT)” defined by a “gamma logarithmic histogram (GLH)”. Third, to verify the algorithm, we analyzed lines of curvature of curved surfaces generated with this system and made these curved surfaces of synthetic resins by numerical control (NC) machining. Through our verification, we were able to confirm that precise curved surfaces were generated early in the modeling phase. Finally, we developed a log-aesthetic curved surfaces generation system that uses VR, because it is difficult to precisely sense a three-dimensional object with a normal display. Furthermore, we used a joystick as an input device that we can operate intuitively by bending and twisting. Our system enabled us to easily create log-aesthetic curved surfaces, to effectively make precise 3D data using CAD, and to accurately evaluate the curved surfaces with VR.

Key words: Log-aesthetic Curved Surface, Computer-Aided Design, Virtual Reality.

1. Introduction
In the field of industrial design, designers create models with computer-aided design (CAD) and evaluate the models using virtual reality (VR) systems. This evaluation requires precise models in the early phases of modeling. However, it is too difficult for designers to control key-lines as they become aesthetic curved surfaces in CAD. Therefore, modelers must modify key-lines of the clay models that are made from the CAD data. These modifications require a great deal of labor.

Many studies have been done on generating aesthetic curves and surfaces. Farin proposed a method for generating control points of a Bézier curve segment of arbitrary degree of monotonic curvature and torsion [1]. The curves generated are called Class A curves. A Class A curve is one type of “log-aesthetic curve.” Here, log-aesthetic curves refer to curves with “logarithmic curvature histograms (LCHs)” (to be described in section 2.1) that are represented by straight lines. Yoshida and Saito proposed a method for generating space curves that showed an arbitrary type of log-aesthetic curve in curvature and torsion by applying the general formula for aesthetic curves [4, 5]. These space curves are called “log-aesthetic space curves.” However, no studies have

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We propose a method for generating an arbitrary aesthetic curved surface using the log-aesthetic space curves. We extend our method to the log-aesthetic space curves and classified the curved surfaces into three types on the basis of our results. Second, we extracted several parameters for controlling characteristics of the log-aesthetic space curves and defined a log-aesthetic curved surface using the log-aesthetic space curves made from these parameters. Here, the log-aesthetic space curve is made from the LCH and LTH. The log-aesthetic curved surface is made from one guide line and two base lines. The base lines are the log-aesthetic space curves, and the curvature and torsion change between the two base lines are controlled by the "control line of distribution of curvature and torsion (CLDCT)" defined with a "gamma logarithmic histogram (GLH)." Third, to verify the algorithm, we analyzed the lines of curvature of the curved surfaces generated with this system and used numerical control (NC) machining to make these curved surfaces out of synthetic resins. We were able to confirm that precise curved surfaces were generated in the early phase of the modeling. Finally, we developed a system that generates log-aesthetic curved surfaces in VR, because it is too difficult for us to precisely sense a third dimension with a conventional flat-screen display. Furthermore, we used a joystick as an input device that we can operate intuitively by bending and twisting. With this system, we were able to easily create log-aesthetic curved surfaces. Therefore, we were able to effectively make precise 3D data using CAD and to accurately evaluate the curved surfaces with VR.

2. Quantitative Analysis Method and Systematization of Curves

2.1 Quantitative Analysis Method

In this section, we give an outline of a quantitative analysis method for the characteristics of a planar curve [2, 3]. We assume that the curve treated in our study, satisfies the following four conditions: 1) plane, 2) open, 3) non-intersecting, and 4) monotone curvature.

We analyzed the design words expert designers use to find what characteristics of a curve they pay attention to. We found some of the words to express the characteristics of a curve. Some words were concerned with curvature changes and the volume of a curve from the viewpoint of mathematics. Here, the term 'volume' is defined as the area bound by the curve and the straight line binding the starting point and the ending point of the curve. Therefore, we define that the characteristics of a curve indicate its curvature changes and volume in this study. In fact, when the curvature changes and volume are defined, the curve is fixed reversely.

We developed a method to analyze curvature changes and the volumes of curves mathematically, simultaneously, and intuitively. In this method, we first interpolate a sample curve using a Bézier curve on a computer. Then, we express the relation between the radius of curvature at every constitutional point on the interpolated curve and the "length" of the curve showing the total length of segmental curves corresponding to this radius of curvature in a log-log coordinate system. This relation is called the LCH. Now, let us explain it in detail in the following (Figure 1).

First, let us denote the total length of the curve by \( S_{all} \) the length of a segmental curve by \( s_j \), the radius of curvature at a constitutional point \( a_i \) by \( \rho_i \), and the interval of the radius of curvature by \( S_{all} \) (the units are mm in all cases.). The radius of curvature \( \rho_i \) at constitutional point \( a_i \) on the curve is obtained by extracting constitutional points \( a_1, a_2, \ldots, a_n \) at equal intervals (e.g., at \( S_{all} = 100 \text{mm} \), the constitutional points were extracted at 0.1mm intervals [in actual dimension], and therefore, \( n = 1000 \), and by calculating the respective radius of curvature \( \rho_1, \rho_2, \ldots, \rho_n \) at the respective constitutional points.

Second, let us denote the interval of the radius of curvature \( \overline{\rho} \) by the interval corresponding to the quotient obtained by dividing the common logarithm \([-3, 2]\) of the value \( \rho_i / S_{all} \) by 0.001, 100] by 100 equally. In other words, \( \overline{\rho}_m = [-3+0.05(m-1), -3+0.05m] \) (\( m \) is an integer between 1 and 100, i.e., \( 1 \leq m \leq 100 \)).

Third, we sum the numbers of occurrences of the common logarithm values of \( \rho_i / S_{all}, \rho_2 / S_{all}, \ldots, \rho_n / S_{all} \) in each interval of \( \overline{\rho} \). From this value (= \( N_j \)), we calculate the length of segmental curve \( s_j \) (= distance between neighboring constitutional points \( x \times N_j \)) in which \( \overline{\rho} \) appears. This means that \( S_{all} = s_1 + s_2 + \ldots + s_{100} \). In addition, we define the "length frequency" \( \tilde{s}_j = [\log_{10}(s_j / S_{all})] \) representing the ratio of the length of the segmental curve to the total length of the curve \( S_{all} \).
The LCH defined above can be obtained by taking $\rho_j$ for the horizontal axis and $s_j$ for the vertical axis, as shown in Figure 1. To draw such a LCH means obtain the locus of $\frac{ds}{d\rho}$ in terms of the interval of the radius of curvature $\rho$ and the length frequency $s$. In addition, the horizontal axis shows the interval of the radius of curvature $\rho$, which is the radius of curvature $\rho$ made dimensionless by dividing by the total length $S_{all}$ of the curve.

In this LCH, the way of the curvature changes is shown by the locus of the $C$ curve, as shown in Figure 1, and the volume of the curve is shown by point A and the distance between points A and B as shown in the same figure. Here, if the distance becomes shorter, the volume of the curve becomes larger.

Furthermore, the "gradient" of the $C$ curve in Figure 1 is defined by

$$\text{"gradient"} = \frac{ds}{d\rho} = \frac{dY}{dX} = \lim \frac{Y_{j-1} - Y_j}{X_{j-1} - X_j} = \lim \frac{s_j - s_{j-1}}{\rho_j - \rho_{j-1}}.$$ 

In this chapter, the term "gradient" means the gradient after transforming the coordinate system to an $X$-$Y$ rectangular coordinate system with the horizontal axis representing $X = \rho$ and the vertical axis representing $Y = s$.

When the "gradient" is $\alpha$, the relation of the interval of the radius of curvature $\rho_j$ and the length frequency $s_j$ is defined by

$$\lim \frac{s_{j-1} - s_j}{\rho_{j-1} - \rho_j} = \ldots = \lim \frac{s_{j-1} - s_j}{\rho_{j-1} - \rho_j} = \alpha.$$ 

Here, if the "gradient" is constant, then the curve has a self-affine property.

We can also derive a LTH describing the curve’s torsion with the same procedure outlined above and assume that the curve satisfies monotonic torsion. In this LTH, the way the torsion changes is shown by the locus of the $T$ curve (i.e., the $T$ curve corresponds to the $C$ curve in the LCH). The gradient of the $T$ curve is called $\beta$.

The characteristics of a space curve are defined with $\alpha$, $\beta$, and the radii of curvature and the radii of torsion at either end of the space curve.

![Figure 1 Logarithmic curvature histogram](image1)

### 2.2 Systematization of Curves

We used this quantitative analysis method on several thousand samples of plane curves of industrial and natural products and studied what characteristics of curves nature and designers control to make aesthetic curves.

As a result of this analysis, we classified these curves into five typical types by differences of the "gradient" of the $C$ curve ($\alpha$) and the "gradient" of the $T$ curve ($\beta$). Practically, we added the circular arc. Consequently, we were able to systematize and classify curves into the six types shown in Figure 2. Here, we call a curve whose $C$ curve or $T$ curve has one straight line a 'monotonic-rhythm curve' and a curve whose $C$ curve or $T$ curve has two straight lines a 'compound-rhythm curve.'

![Figure 2 Classification by logarithmic curvature histogram](image2)
3. Analysis and Classification of Curved Surfaces

We used our analysis method to clarify characteristics of lines of curvature on several curved surfaces of natural and industrial products. The analysis method extracts the lines of curvature on the curved surfaces and draws LCH and LTH from the radii of curvature and radii of torsion of the lines of curvature. We showed that the lines of curvature had characteristics of log-aesthetic space curves and we classified the curved surfaces into three types on the basis of our results: an identical characteristic surface, a transitional characteristic surface, and a composite characteristic surface (Figure 3).

Identical characteristic surface: The lines of curvature have uniform characteristics on one curved surface.

Transitional characteristic surface: The characteristics of lines of curvature gradually change on one side toward the other side.

Composite characteristic surface: The characteristics of lines of curvature change irregularly.

The shapes of curved surfaces differ by a "rhythm of change" of the shapes of lines of curvature between both ends of the curved surface even if the characteristics of lines of curvature at either end of the curved surface are the same. We have a hypothesis that the "rhythm of change" is born of arbitrary uniform transformation of $\alpha$ and $\beta$. We focus on this "rhythm of change" and, in the next section, define some of the parameters that control it.

4. An Algorithm for Generating the Log-aesthetic Curved Surface

4.1 An Algorithm for Generating the Log-aesthetic Space Curve

We used the general formula for log-aesthetic space curves [6] proposed by Yoshida and Saito to generate the log-aesthetic curved surface. With this general formula, we can calculate the curvature and the torsion at each constitutional point on the log-aesthetic space curve and generate a log-aesthetic space curve. This general formula is expressed as follows.

Let us assume that the datum point is an arbitrary point whose $\rho$ and $\mu$ on the curve are anywhere except for 0 or $\infty$. When we assume that the radius of curvature is 1, the radius of torsion is $\nu$, the "gradient" of the $C$ curve is $\alpha$, and the "gradient" of the $T$ curve is $\beta$ on the datum point, the radius of curvature $\rho$ and the radius of torsion $\mu$ of the point on the curve that arc length from a datum point is $s$ are defined by

$$ \rho = \begin{cases} e^{\alpha s} & \text{if } \alpha = 0 \\ \left(\lambda\alpha s + 1\right)^{\frac{1}{2}} & \text{otherwise} \end{cases} $$

(1)

$$ \mu = \begin{cases} e^{(2\pi + \log \nu)} & \text{if } \beta = 0 \\ (\Omega_0 + \nu \beta s + v \nu)^{\frac{1}{2}} & \text{otherwise} \end{cases} $$

(2)

Let us use Eq. (2) in the case of the radius of torsion monotonically increasing for $s$, and Eq. (3) in the case of the radius of torsion monotonically decreasing for $s$.

$$ \mu = \begin{cases} e^{(2\pi + \log \nu)} & \text{if } \beta = 0 \\ (-\Omega_0 + \nu \beta s + v \nu)^{\frac{1}{2}} & \text{otherwise} \end{cases} $$

(3)

Here, it is shown that $s$ is limited by the values of $\alpha$ and $\beta$ [4]. In this paper, we do not treat curves with $s$ that exceeds the limit.

Next, we show a method that uses the general formula to generate the log-aesthetic space curves. Let us assume that $a$ and $b$ are real numbers. When curvature $\kappa(s)$ and torsion $\tau(s)$ ($a \leq s \leq b$) are continuous functions about $s$, a space curve $c(s)$ whose arc length is $s$ exists, and it is shown that the space curve has uniqueness except for
congruent transformation. Let us denote the unit tangent vector by \( t(s) \), the unit principal normal vector by \( n(s) \), and the unit binormal vector by \( b(s) \) on \( c(s) \). When the values are \( c(0), t(0), n(0), \) and \( b(0) \), these functions of the vector satisfy the Frenet-Serret formulas (Eq. (4)) and become the orthonormal basis for an arbitrary value of \( s \).

\[
\begin{align*}
    c'(s) &= t(s) \\
    t'(s) &= \kappa(s)n(s) \\
    n'(s) &= -\kappa(s)t(s) + \tau(s)b(s) \\
    b'(s) &= -\tau(s)n(s)
\end{align*}
\]

(4).

It is shown that \( c(s+\Delta) \) exists in the direction of the tangent vector \( t(s) \) from \( c(s) \). Then \( c(s+\Delta) \) is defined by

\[ c(s+\Delta) = c(s) + \Delta t(s). \] (5).

We can calculate the coordinates of an arbitrary point on \( c(s) \) using Eq. (5) and generate the curve in the system. In this paper, we need to assign the arc length, the radii of curvature, and the radii of torsion of both endpoints on the log-aesthetic space curve. However, we cannot generate the curve because \( \Lambda \) and \( \Omega \) in Eqs. (1), (2), and (3) and the range of \( s \) are unidentified. Also, to use the curves generated in the system easily, we generate the curve from the coordinate origin of the coordinate system. Therefore, let us explain a process to aim the curve. First, let us denote the curve aimed by \( C \), the arc length by \( s \), the radius of curvature of a tip point by \( \rho_s \), the radius of torsion of a tip point by \( \mu_s \), the radius of curvature of an end point by \( \rho_e \), and the radius of torsion of an end point by \( \mu_e \). We assume that the coordinate origin of the coordinate system is a datum point. Second, to generate the curve aimed by \( C \) from the datum point, we normalize each parameter so that \( \rho_s \) becomes 1. It shows that each parameter is divided by \( \rho_s \). Let us then denote the normalized \( \rho_s \) by \( \rho_s^\prime \), the normalized \( \mu_s \) by \( \mu_s^\prime \), the normalized \( \rho_e \) by \( \rho_e^\prime \), the normalized \( \mu_e \) by \( \mu_e^\prime \), the normalized \( s \) by \( \bar{S} \). Third, when Eqs. (1), (2), and (3) are modified for \( \Lambda \) and \( \Omega \), we obtain

\[
\begin{align*}
\Lambda &= \begin{cases} 
\log \rho & \text{if } \alpha = 0 \\
\frac{s}{\rho^\prime - 1} & \text{otherwise}
\end{cases} \\
\Omega &= \begin{cases} 
\log \frac{\bar{S}}{\bar{S}} & \text{if } \beta = 0 \\
\frac{\bar{S} - \bar{S}}{\bar{S}} & \text{otherwise}
\end{cases}
\end{align*}
\]

(6) (7).

Let us use Eq. (7) in the case of the radius of torsion monotonically increasing for \( s \), and Eq. (8) in the case of the radius of torsion monotonically decreasing for \( s \).

\[
\begin{align*}
\Omega &= \begin{cases} 
-\log \frac{\bar{S}}{\bar{S}} & \text{if } \beta = 0 \\
\frac{\bar{S} - \bar{S}}{\bar{S}} & \text{otherwise}
\end{cases}
\end{align*}
\]

(8).

Substituting \( \rho_e^\prime, \mu_e^\prime, \bar{S} \), and \( \bar{S} \) into Eqs. (6), (7), and (8), we obtain

\[
\begin{align*}
\Lambda &= \begin{cases} 
\log \rho & \text{if } \alpha = 0 \\
\frac{\bar{S}}{\rho^\prime - 1} & \text{otherwise}
\end{cases} \\
\Omega &= \begin{cases} 
\log \frac{\bar{S}}{\bar{S}} & \text{if } \beta = 0 \\
\frac{\bar{S} - \bar{S}}{\bar{S}} & \text{otherwise}
\end{cases}
\end{align*}
\]

(9) (10).

Let us use Eq. (10) in the case of the radius of torsion monotonically increasing for \( s \), and Eq. (11) in the case of the radius of torsion monotonically decreasing for \( s \).

\[
\Omega = \begin{cases} 
-\log \frac{\bar{S}}{\bar{S}} & \text{if } \beta = 0 \\
\frac{\bar{S} - \bar{S}}{\bar{S}} & \text{otherwise}
\end{cases}
\]

(11).

We generate a normalized curve from the \( \Lambda \) and \( \Omega \), and obtain the curve aimed by magnifying the normalized curve \( \rho_s \) times. Using this method of generating the log-aesthetic space curve, we are able to generate the log-aesthetic curved surface shown in section 4.2.
4.2. A Definition of a Log-aesthetic Curved Surface

We refer to a figurative process in clay modeling and propose a method for generating a curved surface using one guide line and two base lines that are the log-aesthetic space curves (Figure 4). Let the positions of two base lines be at both endpoints of the guide line. The log-aesthetic curved surface is generated by changing the curvature and the torsion on one base line toward the curvature and the torsion on the other base line gradually with some “rhythm.” So, we must define the rhythm of change of the curvature and the torsion of the base line. When the rhythm of change of the curvature and the torsion on the base line have a self-affine property, we assume that the curved surface generated becomes high quality and aesthetic. The definition of the log-aesthetic curved surface is as follows.

4.2.1. A Definition of the Curves Constituting a Curved Surface

Let us denote the guide line by \( C_g(u) [0 \leq u \leq 1] \), the tangent vector on \( u \) by \( t_g(u) \), the principal normal vector on \( u \) by \( n_g(u) \), the binormal vector on \( u \) by \( b_g(u) \), two base lines by \( C_{b1}(v) \) and \( C_{b2}(v) [0 \leq v \leq 1] \), each tangent vector on \( v \) by \( t_{b1}(v) \) and \( t_{b2}(v) \), each principal normal vector on \( v \) by \( n_{b1}(v) \) and \( n_{b2}(v) \), and each binormal vector on \( v \) by \( b_{b1}(v) \) and \( b_{b2}(v) \). We assume that the start point of \( C_{b1} \) is \( C_g(0) \), the direction of the principal normal vector of \( C_{b1}(0) \) and the direction of the principal normal vector of \( C_g(0) \) are equal (i.e. \( n_{b1}(0) = n_g(0) \)), the direction of the tangent vector of \( C_{b1}(0) \) and the direction of the binormal vector of \( C_g(0) \) are equal (i.e. \( t_{b1}(0) = b_g(0) \)), the start point of \( C_{b2} \) is \( C_g(1) \), the direction of the principal normal vector of \( C_{b2}(0) \) and the direction of the principal normal vector of \( C_g(1) \) are equal (i.e. \( n_{b2}(0) = n_g(1) \)), and the direction of the tangent vector of \( C_{b2}(0) \) and the direction of the binormal vector of \( C_g(1) \) are equal (i.e. \( t_{b2}(0) = b_g(1) \)). Then, we can generate the log-aesthetic curved surface by getting the curve \( C_u(v) \) that interpolates base lines starting from the point \( C_g(u) \). Here, let us consider the curvature at each point on the curve \( C_u \). We assume that the curvature on \( C_{b1}(v) \) is \( \kappa_{b1}(v) \), the curvature on \( C_{b2}(v) \) is \( \kappa_{b2}(v) \), and the curvature on \( C_u(v) \) is \( \kappa_u(v) \). We calculate \( \kappa_u(v) \) using \( \kappa_{b1}(v) \) and \( \kappa_{b2}(v) \). However, we assume that the total lengths of \( C_{b1} \), \( C_{b2} \), and \( C_u \) are identical. As a limit, using a “gamma logarithmic curvature histogram (to be described in section 4.2.2)”, \( \kappa_u(v) \) monotonically increases for \( u \). In the same way, the torsion of \( C_u(v) \) is defined to be \( \tau_u(v) \) (Figure 5). As a limit, \( \tau_u(v) \) monotonically increases for \( v \).

We can define the shape of \( C_u \) by getting \( \kappa_u(v) \) and \( \tau_u(v) \).
4.2.2. A Definition of the Log-aesthetic Curved Surface

To express the rhythm of change of $\kappa_u(v)$ for $u$ when we assume $v$ uniformity, we define a gamma logarithmic curvature histogram (GLCH) by applying the LCH. Let us consider a virtual curve with the same curvature change for $\kappa_u(v)$ as for $u$. We call this virtual curve a control line of distribution of curvature and torsion (CLDCT). Let us denote CLDCT on $v$ by the CLDCT($u$). By a method that is similar to that for finding the radii of curvature on the curve from LCH, we can calculate $\kappa_u(v)$ of an arbitrary point on the CLDCT($u$) with the largest curvature $\kappa_u(v)$ on the tip point and $\kappa_u(v)$ on the end point on the CLDCT($u$). And, we change the intervals of $u$ and $v$ of $\kappa_u(v)$ \([0\leq v\leq 1]\) depending on the precision necessary and calculate the value of $\kappa_u(v)$. We generate the curve using $\kappa_u(v)$ \([0\leq v\leq 1]\) on $C_u$. When we generate the CLDCT($u$), we need the total length of its curve $S(v)$. So, we consider that $S(v)$ is approximated with the length of the straight line connecting $C_b1(v)$ and $C_b2(v)$. And, we call the histogram calculating $\kappa_u(v)$ a GLCH and the line drawn by linking the tops of the histogram a “$\gamma_c$ curve.” In the same way, we call a “gamma logarithmic torsion histogram (GLTH)” and a “$\gamma_t$ curve” for torsion. Finally, we calculate $\kappa_u(v)$ and $\tau_u(v)$ on $u$ and $v$ and generate the curved surface. When the $\gamma_c$ curve and the $\gamma_t$ curve become a straight line (i.e. when the changes of the curvature and the torsion on the CLDCT($u$) have a self-affine property), we define the generated curved surface as a log-aesthetic curved surface, and the gradients of the $\gamma_c$ curve and the $\gamma_t$ curve are $\gamma_c$ and $\gamma_t$. We can control the characteristics of log-aesthetic curved surfaces by changing the values of $\gamma_c$ and $\gamma_t$ (Figure 6).

![Figure 6 Gamma logarithmic curvature histogram and gamma logarithmic torsion histogram](image)

4.2.3 An Algorithm for Generating the Log-aesthetic Curved Surface

The log-aesthetic curved surface can be generated by following the process of the definition outlined above. Concretely, we assign the start point of the generated curved surface, the shape of a guide line and two base lines, the generating direction of the guide lines, and $\gamma_c$ and $\gamma_t$. Next, we control values of $\kappa_u(v)$ and $\tau_u(v)$ \([0\leq v\leq 1]\) by CLDCT($u$) defined with $\gamma_c$ and $\gamma_t$, and we calculate $\kappa_u(v)$ and $\tau_u(v)$ on $C_d(v)$. The radius of curvature and the radius of torsion on a start point of CLDCT($u$) are equal to those on $C_d(v)$. The radius of curvature and the radius of torsion on an end point of CLDCT($u$) are equal to those on $C_d(v)$. CLDCT($u$) is defined with the total length of the curve of CLDCT($u$), $\gamma_c$, and $\gamma_t$. The curvature and the torsion on $u$ and $v$ of CLDCT($u$) are equal to $\kappa_u(v)$ and $\tau_u(v)$. Therefore, we can calculate $\kappa_u(v)$ and $\tau_u(v)$ on $C_d(v)$ from CLDCT($u$) by applying the algorithm for generating the log-aesthetic space curve outlined in section 4.1. When the generating direction on $C_d(0)$ is the binormal direction on $C_d(u)$, we calculate the coordinates of all constitutional points of each $C_d$ with $\kappa_u(v)$, and $\tau_u(v)$ \([0\leq v\leq 1]\) \([0\leq u\leq 1]\) by turns (Figure 7, Step 1). In this paper, we express the log-aesthetic curved surface in the system by generating triangular polygons among these constitutional points calculated (Figure 7, Steps 2 and 3). The above is the algorithm for generating the log-aesthetic curved surface on the system. In addition, we can divide distances between constitutional points into 2–10 intervals per unit length if necessary.
4.3 A System for Generating the Log-aesthetic Curved Surface

We developed a system for generating log-aesthetic curved surfaces using the method proposed (Figure 8). The system has the following features.
- We can generate the log-aesthetic curved surface by inputting parameters as shown in figure 8 almost in real time.
- We can save the generated curved surface in OBJ format and edit it with general CAD software.
- We can show the base line, the radius of curvature, and the radius of torsion at each point, the tangent vector, the principal normal vector, the binormal vector on guide lines, and the zebra mapping of the curved surface.

4.4 Evaluation of the Availability of the Generated Curved Surface

We evaluated samples of curved surfaces generated by this system. First, we generated the log-aesthetic curved surfaces of $\gamma_c=2.0$ and $\gamma_c=-2.0$. We show the zebra mapping of these curved surfaces in Figure 9. Here, the two curved surfaces are identical in all parameters except for $\gamma_c$. As a result, the log-aesthetic curved surface of $\gamma_c=2.0$ suddenly changes in comparison with that of $\gamma_c=-2.0$. The log-aesthetic curved surface of $\gamma_c=2.0$ appears to have more volume than that of $\gamma_c=-2.0$.

Second, we analyzed how the characteristics of the base lines $C_\alpha$ changed. We set “$\alpha$”s of the C curves of base lines of both ends to 0.666 and -0.666 and generated the curved surface. The LCHs of base lines $C_\alpha$ when $\gamma_c=1.0$ and when $\gamma_c=-1.0$ are shown in Figure 10. We confirmed that the characteristics of base lines $C_\alpha$ change from one side to the other side gradually. In addition, the neutral point (horizontal gradient) of the gradient of the C curve when $\gamma_c=-1.0$ is closer to the $C_{b1}$ side than when $\gamma_c=1.0$.

Third, we generated the log-aesthetic curved surface of $\gamma_c=1.0$ and $\gamma_c=-1.0$, and we analyzed the characteristics of the lines of curvature of these curved surfaces that are identical in all parameters except for $\gamma_c$ (Figures 11 and 12). We found that the characteristics of the lines of curvature of both sides are almost the same as those of the base lines. We were able to control the characteristics of the curved surfaces by controlling the characteristics of the base lines.

We also created two log-aesthetic curved surfaces made with synthetic resin models by using NC machining. Several people experienced in shape design visually inspected the quality of these log-aesthetic curved surfaces (Figure 13 and 14). Furthermore, an inspection of highlights of the log-aesthetic curved surface indicated that the highlights were similar to those calculated by the system.

As a result, we found the algorithm and the generated log-aesthetic curved surfaces to be useful for modeling in figurative processes.
5. Development of a Log-aesthetic Curved Surface Generation System with VR

We developed a system that generates log-aesthetic curved surfaces using VR because it was too difficult for us to precisely sense three-dimensional (3D) objects with a conventional flat-screen display. Furthermore, we used a joystick as an input device that we could operate intuitively by bending and twisting.

We compared each curved surface displayed on three types of VR devices (a 200-in. 3D display [Figure 15], a 22-in. 3D display, and a 24-in. normal display) using a curved surface made with NC machining. Through this comparative evaluation, we can found three important points about evaluation using VR.

1) Using VR is effective for evaluating the curved surfaces. The appearances of the curved surface displayed by each of the three types of devices differ somewhat from each other. Through comparison with the actual curved surface, we must investigate the display properties of these devices in advance.

2) We can see the shapes of the curved surfaces more easily when the displayed curved surfaces are rotated and magnified to several times the actual size.

3) It may be necessary to change the colors of the curved surfaces and the positions of the lights depending on the situation to better see the shapes of the curved surfaces.
6. Conclusion
The following results were obtained in this research.
1) We proposed a method of analyzing the characteristics of curved surfaces of natural and industrial products by LCH and LTH.
2) We used this method to analyze the characteristics of curved surfaces of natural and industrial products and classified the curved surfaces into three types on the basis of our results: an identical characteristic surface, a transitional characteristic surface, and a composite characteristic surface.
3) We extracted several parameters for controlling characteristics of the log-aesthetic space curves and defined the log-aesthetic curved surface using the log-aesthetic space curves made from these parameters. Furthermore, we developed a system that generates log-aesthetic curved surfaces using VR. With this system, we were able to create log-aesthetic curved surfaces easily. Therefore, we were able to make precise 3D data effectively with CAD and accurately evaluate curved surfaces with VR.

The following subjects remain as future work.
1) It is not yet clarified what value of $\gamma_c$ and $\gamma_t$ in the curved surface of other natural and industrial products. It is necessary to analyze the characteristics of an expanded range of sample products.
2) It is necessary to systematize the characteristics of the log-aesthetic curved surfaces with $\gamma_c$ and $\gamma_t$ because there are many kinds of combinations used for generating log-aesthetic curved surfaces.
3) It is necessary to improve the interface of the editor for generating the log-aesthetic curved surfaces. For example, an interface on which the lines of curvature are displayed and evaluated on the curved surfaces in real time would enable us to edit the curved surfaces intuitively.

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